AFRL-SR-AR-TR-04-

0528

REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE 09/24/04

3. REPORT TYPE AND DATES COVERED Final Technical Report 02/01/2001 through 06/30/2004

4. TITLE AND SUBTITLE
Non-Deteriorating Numerical Methods and Artificial Boundary Conditions for the Long-Term Integration of Maxwell's Equations
F49620-01-1-0187

5. FUNDING NUMBERS
F49620-01-1-0187

6. AUTHOR(S)
Semyon T. Tsynkov

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
North Carolina State University
Center for Research in Scientific Comp.
2610 Stinson Drive
Harrelson
Raleigh, NC 27695-8205

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Dr. Arje Nachmann
AFOSR/NM
4015 Wilson Blvd. Rm 713
Arlington, VA 22203

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views opinions and /or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT ***(Maximum 200 Words)***

All the original objectives of the project have been addressed. Lacunae have been identified and studied theoretically in the solutions of the Maxwell's equations that govern the propagation of electromagnetic waves in vacuum. The case of acoustics has also been analyzed along the same lines. A lacunae-based numerical algorithm has been built for the long-term integration of the Maxwell's equations. Its key property is the grid convergence that is uniform in time; in other words, there is no error build-up during the integration over arbitrary long intervals. Lacunae-based artificial boundary conditions (ABCs) have also been constructed and tested; their actual performance fully meets the theoretical expectations. The corresponding results have been presented at scientific conferences and published in peer reviewed journals.
New theoretical and numerical issues have also been discovered in the course of the project that were not known at the time of its inception. They have warranted the extension of the project by six months beyond its original expiration date. It turns out that implementation of the lacunae-based ABCs for the Maxwell's equations necessitates that special auxiliary field sources be used in the form of solenoidal currents and zero charges. Particular types of such sources have been analyzed and employed previously. A general procedure for their construction has been introduced and implemented at the most recent stage of the project. The results of this implementation will be reported in a forthcoming paper.

14. SUBJECT TERMS

15. NUMBER OF PAGES 7

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

20041028 079

*** AFOSR Grant # 49620-01-1-0187**

Final (Cumulative) Performance Report
01 Feb 2001 -- 30 Jun 2004
Submitted in September 2004

PI Name: *Semyon Tsynkov*
PI Address: Center For Research in Scientific Computation
North Carolina State University
Box 8205
Raleigh, NC 27695
PI Phone: 919-515-1877
PI Fax: 919-515-1636
PI email address: tsynkov@math.ncsu.edu

[List Co-PIs/Subcontractors, if any]

*** OBJECTIVES**
[List the original objectives (preferably in bulletized form) of the research effort or the statement of work. State new or revised objectives if they have changed and the reason why.]

The overall objective of the project is to extend the previously obtained results on non-deteriorating numerical integration of, and lacunae-based artificial boundary conditions (ABCs) for, the scalar 3D wave equation, to the case of vector wave-type models (i.e., systems of equations), with the primary emphasis on the Maxwell's equations. This includes:

1. Identifying the lacunae (areas of zero solution behind sharp aft fronts of the waves) of a given hyperbolic system (Maxwell's equations).

2. Constructing a lacunae-based non-deteriorating algorithm for the long-term numerical integration of the corresponding system, proving its temporally uniform grid convergence, and demonstrating it experimentally.

3. Constructing the lacunae-based ABCs and experimentally studying their performance for a variety of cases.

*** STATUS OF EFFORT**
[A brief statement of progress towards achieving the research objectives. Please make this substantive (Limit to 200 words).]

All the original objectives of the project have been addressed. Lacunae have been identified and studied theoretically in the solutions of the Maxwell's equations that

govern the propagation of electromagnetic waves in vacuum. The case of acoustics has also been analyzed along the same lines. A lacunae-based numerical algorithm has been built for the long-term integration of the Maxwell's equations. Its key property is the grid convergence that is uniform in time; in other words, there is no error build-up during the integration over arbitrary long intervals. Lacunae-based artificial boundary conditions (ABCs) have also been constructed and tested; their actual performance fully meets the theoretical expectations. The corresponding results have been presented at scientific conferences and published in peer reviewed journals.

New theoretical and numerical issues have been discovered in the course of the project that were not known at the time of its inception. They have warranted the extension of the project by six months beyond its original expiration date. It turns out that implementation of the lacunae-based ABCs for the Maxwell's equations necessitates that special auxiliary field sources be used in the form of solenoidal currents and zero charges. Particular types of such sources have been analyzed and employed previously. A general procedure for their construction was introduced and implemented at the most recent stage of the project. The results of this implementation will be reported in a forthcoming paper.

*** ACCOMPLISHMENTS/NEW FINDINGS**

[Describe research highlights, their significance to the field, their relationship to the original goals, their relevance to the AF's mission, and their potential applications to AF and civilian technology challenges.]

The structure of lacunae in the solutions of the Maxwell's equations driven by compactly supported sources has been analyzed for the case of the propagation of electromagnetic waves in vacuum. Similar analysis has been conducted for the linearized Euler's equations that govern the propagation of acoustic waves. In either case, the sources of waves can engage in an accelerated motion.

In general, the phenomenon of lacunae, or propagation with no after effects, is known to take place only when the number of space dimensions is odd. There are additional, rather subtle, conditions that tell between the sharp aft fronts and the diffused ones. Altogether, the diffusive propagation is common and lacunae are rare. However, for the two aforementioned systems, Maxwell and acoustics, they do exist, provided that the source terms satisfy some constraints. The latter have been studies thoroughly and found not overly restrictive but not limitation-free either, like in the case of the scalar d'Alembert equation.

Lacunae-based numerical algorithms have been designed for the long-term integration of the foregoing systems of equations driven by continuously operating compact sources. The key property of these methods is temporally uniform grid convergence; it has been proven theoretically. In practice, it provides for no deterioration in the solution quality over arbitrary long time intervals.

Lacunae-based ABCs have been constructed and implemented for systems. The key difference compared to the scalar case is that the auxiliary sources (right-hand sides) used for this construction must satisfy certain constraints that enable the presence of lacunae. In particular, for the Maxwell's system, the corresponding auxiliary currents must be divergence-free. Existence of such currents in a general setting is, in fact, an independent important issue that requires attention, see below. Performance of the lacunae-based ABCs was tested experimentally; the design convergence rate of the algorithm has actually been demonstrated over long time intervals.

In the meantime, all the codes used in the simulations employ cylindrical symmetry. This still allows one to capture the essential three-dimensional effects by means of relatively inexpensive computations with only two independent spatial variables.

Lacunae-based ABCs guarantee the complete transparency of the outer boundary for all the outgoing waves.

They are obtained directly for the discrete approximation of the original continuous problem, and can be used with any finite-difference scheme.

The extent of temporal nonlocality of these genuinely time-dependent ABCs is fixed and limited.

The ABCs can handle curvilinear boundaries on regular Cartesian grids with no adaptation required.

The ABCs allow for an accelerated motion of the sources of waves, a capability that, to the best of the PI's knowledge, has not been attained previously.

It has also been determined that in a number of specially designed "harsh" test cases, when the quality of the discrete lacunae was intentionally made suboptimal, the performance of the ABCs, although deteriorated in a predictable way, was much better on finer grids than on coarse grids. To some extent this observation is counterintuitive, because a typical instability would rather manifest itself by a rapid deterioration of the solution when the grid is refined. As of yet, only a qualitative explanation of this phenomenon can be offered, rather than a rigorous theoretical insight. This will be a subject for the future study.

Perhaps, one of the most important general findings in the course of this project is that compared to the scalar case, design of the lacunae-based methods for systems brings about a number of issues to be addressed. First and foremost, those have to do with making sure that the lacunae (i.e., sharp aft fronts of the waves) do exist in the solutions of interest. For the vital case of the 3D Maxwell's system, an obstacle is presented by the necessary continuity requirement that its source terms (currents and charges) must satisfy.

Recently, a fundamental general result has been proven on existence of the special auxiliary sources for the Maxwell system (solenoidal currents) that would satisfy the continuity equation identically. Its proof employs Whitney-type extensions. In the aforementioned computer implementations of the lacunae-based ABCs for the Maxwell's equations, the required solenoidal currents were obtained under some specific assumptions with the help of the vector potential. A constructive universal procedure for obtaining those solenoidal auxiliary currents has also been introduced most recently and implemented in the code. It is based on Taylor expansions. The corresponding results are to be reported in a forthcoming publication.

*** PERSONNEL SUPPORTED**

[List professional personnel supported by and/or associated with the research effort.]

S. Tsynkov - PI.

*** PUBLICATIONS**

[List peer-reviewed publications submitted and/or accepted during the 12-month period starting the previous 1 August (or since start for new grants).] **- for the entire duration of the grant:**

V. S. Ryaben'kii, S. V. Tsynkov, and V. I. Turchaninov, *"Long-time numerical computation of wave-type solutions driven by moving sources,"* Applied Numerical Mathematics, 38 (2001) 187-222.

V. S. Ryaben'kii, S. V. Tsynkov, and V. I. Turchaninov, *"Global discrete artificial boundary conditions for time-dependent wave propagation,"* J. Comput. Phys, 174 (2001), pp. 712-758.

S. V. Tsynkov, *"Artificial Boundary Conditions for the Numerical Simulation of Unsteady Acoustic Waves,"* J. Comput. Phys., 189 (2003) pp. 626-650.

S. V. Tsynkov, "Artificial Boundary Conditions for the Numerical Simulation of Unsteady Electromagnetic Waves," Center for Research in Scientific Computation, North Carolina State University, Tech. Report No.~CRSC--TR03--19, Raleigh, NC, April 2003.

S. V. Tsynkov, *"Lacunae-Based Artificial Boundary Conditions for the Numerical Simulation of Unsteady Waves Governed by Vector Models,"* in: *Mathematical and Numerical Aspects of Wave Propagation WAVES 2003,* Proceedings of the Sixth International Conference Held at Jyvaskyla, Finland, June 30 - July 4, 2003, G. C. Cohen, E. Heikkola, P. Joly, and P. Neittaanmaki, eds., Springer, Berlin, 2003, pp.~103--108.

S. V. Tsynkov, *"On the Application of Lacunae-Based Methods to Maxwell's Equations,"* J. Comput. Phys., 199 (2004) pp. 126-149.

A. E. Chertock and S. V. Tsynkov, *"On the Application of Lacunae-Based Methods to Maxwell's Equations. Part II,"* in progress.

*** Invited papers presented at conferences:**

"Non-Deteriorating Numerical Methods and Artificial Boundary Conditions for Unsteady Wave Propagation," presented in June 2001, in Uppsala, Sweden, at the Fifth International Conference on Spectral and High Order Methods (ICOSAHOM'01), as a part of the special minisymposium to honor Saul Abarbanel on the occasion of his 70th birthday.

"Long-Term Numerical Solution of Unsteady Wave Propagation Problems," presented at 2002 AFOSR Electromagnetics Conference, San Antonio, TX, January 2002.

"Artificial Boundary Conditions for the Simulation of Unsteady Waves Governed by Vector Models," presented at 2003 AFOSR Electromagnetics Conference, San Antonio, TX, January 2003.

"Artificial Boundary Conditions for the Simulation of Unsteady Waves Governed by Vector Models," presented at American Mathematical Society Eastern Sectional Meeting, New York, NY, April 12-13, 2003.

"The Huygens' Principle in Computational Electromagnetics and Beyond," presented at 2004 AFOSR Electromagnetics Conference, San Antonio, TX, January 2004.

*** INTERACTIONS/TRANSITIONS**
*** Participation/Presentations At Meetings, Conferences, Seminars, etc.**
[Be selective, but be sure to include participations that reflect the quality/impact of the effort]

"Lacunae-Based Artificial Boundary Conditions for the Numerical Simulation of Unsteady Waves Governed by Vector Models," presented at the Sixth International Conference on Mathematical and Numerical Aspects of Wave Propagation, Jyvaskyla, Finland, June 30 - July 4, 2003.

*** INVITED SEMINARS:**

Colloquia/Seminars at the University of California, Davis, University of California, Berkeley, NASA Ames Research Center, Columbia University, Courant Institute of Mathematical Sciences, Brown University, Northwestern University, University of Maryland, University of Southern California, Illinois Institute of Technology, Indiana University - Purdue University (Indianapolis), University of North Carolina (Charlotte),

Duke University, University of Connecticut, Stanford University, Tel Aviv University (Israel), Keldysh Institute for Applied Mathematics (Moscow, Russia).

*** Consultative And Advisory Functions To Other Laboratories And Agencies**
[Consultative and advisory functions to other laboratories and agencies, especially Air Force and other DoD laboratories. Provide factual information about the subject matter, institutions, locations, dates, and name(s) of principal individuals involved.]

*** Transitions**
[Describe cases where knowledge resulting from your effort is used, or will be used, in a technology application. Transitions can be to entities in the DOD, other federal agencies, or industry. Briefly list the enabling research, the laboratory or company, and an individual (with contact information) in that organization who made use of your research.]

*** NEW DISCOVERIES, INVENTIONS, OR PATENT DISCLOSURES**
[If none, report None.]

None

*** HONORS/AWARDS**
[List honors and awards (include dates) received during the grant/ contract period. List lifetime achievement honors (include dates) such as Nobel prize, honorary doctorates, and society fellowships prior to his effort.]

Doctor of Science (Habilitation) in Computational Mathematics - an advanced post-PhD degree, awarded to S. Tsynkov by the Russian Academy of Sciences on May 14, 2004, - upon submission of a dissertation and a public defense.